ARC LENGTH OF A CIRCLE

# Definition

Arc length is better defined as the distance along the part of the circumference of any circle or any curve (arc). Any distance along the curved line that makes up the arc is known as the arc length.

# Arc Length Formula

The arc length of a circle can be calculated using different formulas, based on the unit of the center angle of the arc. The arc length formula in radians can be expressed as,

Arc Length = θ × r

L = Arc Length

θ = Center angle of the arc in radians

r = Radius of the circle

# How to Find Arc Length of a Curve?

The arc length of an arc of a circle can be calculated using different methods and formulas based on the given data. Some important cases are given below,

Find arc length with the radius and central angle

Find arc length without the radius

Find arc length without the central angle

## How to Find Arc Length with the Radius and Central Angle?

The arc length of a circle can be calculated with the radius and central angle using the arc length formula,

Length of an Arc = θ × r, where θ is in radian.

Length of an Arc = θ × (π/180) × r, where θ is in degree.

## How to Find Arc Length without the Radius?

The arc length of a circle can be calculated without the radius using:

### Central angle and the sector area:

Multiply the sector area by 2 and further, divide the result by the central angle in radians.

Find the square root of the result of the division.

Multiply this obtained root by the central angle again to get the arc length.

The units of this calculated arc length will be the square root of the sector area units.

**Example:** Calculate the arc length of a curve with sector area 25 square units and the central angle as 2 radians.

We have,

Sector area = 25 units

Central angle = 2 radians

Step 1: Sector area × 2 = 25 × 2 = 50

Step 2: 50/central angle = 50/2 = 25

Step 3: √25 = 5

Step 4: 5 × central angle = 5 × 2 = 10 units

Thus, arc length = 10 units

### Central angle and the chord length:

Divide the central angle in radians by 2 and further, perform the sine function on it.

Divide the given chord length by twice the result of step 1. This calculation gives you the radius as result.

Multiply the radius by the central angle to get the arc length.

**Example:** Calculate the arc length of a curve, whose endpoints touch a chord of the circle measuring 5 units. The central angle subtended by the arc is 2 radians.

We have,

Chord length = 5 units

Central angle = 2 radians

Step 1: Central angle/2 = 2/2 = 1

Step 2: Sin(1) = 0.841

Step 3:Chord length/ (2 × 0.841) = 5/ 1.682 = 2.973 units = radius

Step 4: Arc length = radius × central angle = 2.973 × 2 = 5.946 units

Thus, arc length = 5.946 units

## How to Find Arc Length Without the Central Angle?

The arc length of a circle can be calculated without the angle using:

### Radius and the sector area:

Multiply the sector area by 2.

Then divide the result by the radius squared (the units should be the same) to get the central angle in radians.

Multiply the central angle by the radius to get the arc length.

**Example:** Calculate the arc length of a curve with sector area 25 square units and radius as 2 units.

We have,

Sector area = 25 units

Central angle = 2 units

Step 1: Sector area × 2 = 25 × 2 = 50

Step 2: 50/radius2 = 50/4 = 12.5 = central angle(rad)

Step 3: Arc length = radius × central angle = 2 × 12.5 = 25 units

Thus, arc length = 25 units

### Radius and chord length:

Divide the chord length by twice the given radius.

Find the inverse sine of the obtained result.

Double the result of the inverse sine to get the central angle in radians.

Multiply the central angle by the radius to get the arc length.

**Example:** Calculate the arc length of a curve, whose endpoints touch a chord of the circle measuring 5 units. The radius of the circle is 2 units.

We have,

Chord length = 5 units

Central angle = 2 units

Step 1: Chord length/(2 × radius) = 5/(2 × 2) = 1.25

Step 2: Sin-1(1.25) = 0.949

Step 3: Central angle = 2 × 0.949 = 1.898 radians

Step 4: Arc length = radius × central angle = 2 × 1.898 = 3.796 units

Thus, arc length = 3.796 units